Physics 325 Homework 11

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Due: Wednesday 12/4/02 by 4 PM

1) A wave form

$$f(x) = 50 \sin \frac{3\pi x}{L} \cos \frac{\pi x}{L}$$

is defined between $x = 0$ and $x = L$. Determine the coefficients a_n and b_n if we represent $f(x)$ as:

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L})$$

Hint: this problem is much easier if you first use trig identities to write $f(x)$ as a sum of sines and cosines.

2) The function $f(x)$ is defined over a region $2L$ as:

$$f(x) = \begin{cases} -h & -L < x < 0 \\ +h & 0 < x < L \end{cases}$$

Find the Fourier series coefficients for $f(x)$. Plot the first nine coefficients vs. the integer m.

3) A string is stretched between two fixed posts ($x = 0$ and $x = L$). It has initial displacement $y(x, 0) = 0$, and initial velocity $\dot{y}(x, 0) > 0$. For $t > 0$, $y(x, t)$ is given by:

$$y(x,t) = \sum_{n=1}^{\infty} (A_n \cos \frac{n\pi vt}{L} + B_n \sin \frac{n\pi vt}{L}) \sin \frac{n\pi x}{L}$$

where v is the phase velocity of the waves on the string.

a) Derive an equation for B_n in terms of $\dot{y}(x, 0)$.

b) Suppose that

$$\dot{y}(x, 0) = v_o \left[\sin \frac{\pi x}{L} + \sin \frac{2\pi x}{L} \right].$$

Find the B_n for $n = 1, 2, ..., \infty$.

c) Find $y(x, t)$ at $t = L/2v$ using the formulae for the B_n found in part b).