**Physics 325 Homework 4**

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**Due: 10/2/02 by 430 PM**

**Problem 1 - Spinors**

The three Pauli spin matrices (aka "spinors") are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that:

a) $\sigma_i^2 = 1$ for $i = (1,2,3)$

b) $\sigma_i \sigma_j = i\sigma_k$ for cyclic permutation of $(i,j,k) = [(1,2,3),(2,3,1),(3,1,2)]$

c) $\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij}$ where $\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$

d) pairs anticommute, that is: $\sigma_i \sigma_j = -\sigma_j \sigma_i$ for i $\neq$ j

e) $\sigma_i \sigma_j - \sigma_j \sigma_i = 2i\sigma_k$ for i $\neq$ j

**Problem 2 - Half Life**

The half life of a radioactive substance is the time it takes for half of it to (exponentially) decay. Suppose a radioactive sample consists of components A and B with half-lives 2 and 3 hours, respectively. Assume that the decay products are gases which escape at once. At the end of 12 hours, the sample weighs 56 grams, and at the end of 18 hours, it weighs 12 grams. Use Cramer's Rule to find the amounts of A and B that were originally present.

Hint: The exponential decay equation is:

$$N(t) = N_o e^{-t/\tau}$$

where $\tau$ is the decay constant. How is $\tau$ related to the half life?

**Problem 3.**

Show that the relation between two sets of polar coordinates $(x, y)$ and $(x', y')$ shown in the attached figure can be given by:

$$x' = x\cos\theta + y\sin\theta \quad \text{and} \quad y' = x\sin\theta + y\cos\theta$$

where $\theta$ is the angle of rotation between the two sets of coordinates.