Physics 325 Homework 5

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Due: 10/16/02 by 4 PM

Problem 1 - Photon Gas

A photon gas is characterized by

$$\Omega(E) = \exp\left[\frac{4}{3}\left(\frac{\pi^2}{15\hbar^3 c^3} V E^3\right)^{1/4}\right] \delta E$$

where δE is small compared to the exponential.

a) If the entropy S is given by $S = k \ln \Omega$ find the entropy of the system in terms of E and V (and various constants). (Hint: you can neglect the term $k \ln \delta E$.)

b) Find the entropy of the system in terms of T and V if the temperature is given by:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$$

c) Find the pressure of the system in terms of T and V if the pressure is given by:

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{E,V}$$

.

d) Find the energy of the system in terms of P and V.

Problem 2 - Spin system

The specific magnetization of a system is defined as

$$M \equiv -\frac{1}{V}\left(\frac{\partial E}{\partial H}\right)_{N,S}$$

For a spin system, the volume can be considered to be a fixed parameter and the energy is defined as:

$$E = -N\mu H \tanh\left(\frac{\mu H}{kT}\right)$$

For isentropic changes to this system, $\frac{E}{N\mu H}$ = constant. (Isentropic means that the entropy stays constant.)

a) Show that $M(E, N, H) = -E/HV$.

b) Find $M(T, N, H)$.

c) The isothermal susceptibility is defined by

$$\chi_T \equiv \left(\frac{\partial M}{\partial H}\right)_{N,T}$$

Compute χ_T for the spin system.

d) The adiabatic susceptibility is defined by

$$\chi_S \equiv \left(\frac{\partial M}{\partial H}\right)_{N,S}$$

Compute χ_S for the spin system.

Problem 3 - Particle in a box

Consider a particle confined within a box in the shape of a rectangular parallelpiped of edges L_x, L_y, and L_z. The possible energy levels of this particle are then given by

$$E = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$$

where n_x, n_y, n_z = 1,2,3...

Now suppose that the particle is in a given state specified by particular values of the three integers n_x, n_y, and n_z.

a) Find the amount of work done δW_x when the particle presses on the box with a force F_x, expanding it by a small amount δL_x

b) The force exerted by the wall on the particle F_2 is equal and opposite to the force the particle exerts on the wall F_1. By considering how the energy of the particle's state changes when the length L_x of the box is changed by a small amount dL_x, show that the force exerted by the particle in this particular state on a wall perpendicular to the x axis is given by $F_1 = -\partial E/\partial L_x$.

c) By dividing by the area of the wall, calculate explicitly (by taking $\partial E/\partial L_x$) the force per unit area (or pressure) on this wall. (Hint: what plane does this particular wall occupy?) The pressure found in this manner is the mean pressure $< P_x >$ on the wall normal to x.

d) Show that the mean pressure can be very simply expressed in terms of the mean energy $< E >$ of the particle and the volume $V = L_x L_y L_z$ of the box, and derive the expression relating $< P >$ to $< E >$. (Hint: how do $< P_x >$, $< P_y >$ and $< P_z >$ relate to $< P >$?.