Physics 325 Homework 9

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Due: 11/13/02 by 4 PM

Problem 1 - Two moving charges

An electric charge q_1 moving with velocity $\vec{v}_1$ produces a magnetic induction $\vec{B}$ given by:

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q_1}{r^2} \vec{v}_1 \times \vec{r}_o$$

where $\vec{r}_o$ points from q_1 to the point at which $\vec{B}$ is measured (Biot-Savart Law)

a) Show that the magnetic force on a second charge q_2 with velocity $\vec{v}_2$ is given by the triple vector product:

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q_1 q_2}{r^2} \vec{v}_2 \times (\vec{v}_1 \times \vec{r}_o)$$

b) What is the corresponding magnetic force $\vec{F}_1$ that is exerted by q_2 on q_1? (Be sure to define your unit radial vector.) How do F_1 and $\vec{F}_2$ compare?

c) What are $\vec{F}_1$ and $\vec{F}_2$ if $\vec{v}_1$ and $\vec{v}_2$ have parallel trajectories (i.e. particles q_1 and q_2 are moving along side by side)?

Problem 2 - Boas 6.6.11 and 12

11a) Given $\phi = x^2 - y^2$ sketch on one graph the curves $\phi = 4$, $\phi = 1$, $\phi = 0$, $\phi = -1$, $\phi = -4$. If ϕ is the electrostatic potential, the curves $\phi=$ const. are equipotentials, and the electric field is given by $\vec{E} = -\nabla\phi$. If ϕ is temperature, the curves $\phi=$ const. are isothermals and $\nabla\phi$ is the temperature gradient, heat flows in the directions $-\nabla\phi$.

11b) Find and draw on your sketch the vectors $-\nabla\phi$ at the points $(x, y) = (\pm1, \pm1), (0, \pm2), (\pm2, 0)$. Then remembering that $\nabla\phi$ is perpendicular to $\phi=$ const., sketch without computation, several curves along which heat would flow.

12a) For the above, find the magnitude and direction of the electric field at $(2, 1)$

12b) Find the direction in which the temperature is decreasing most rapidly at $(-3, 2)$

12c) Find the rate of change of temperature with distance at $(1, 2)$ in the direction $3\hat{\imath} - \hat{\jmath}$.

Problem 3 - Electric Field

The electrostatic field of a point charge q is given by:

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

Use the divergence in spherical polar coordinates to calculate $\nabla \cdot \vec{E}$. What happens at the origin?