Introduction
Busy educators sometimes have trouble finding ways to help their students feel the excitement of science in action. As a part of its educational effort, the NASA Education and Public Outreach (E/PO) group at Sonoma State University (SSU) has put together a series of activities based on the science of one of NASA’s exciting space missions: the Gamma-ray Large Area Space Telescope (GLAST).

On these pages you will find an introduction to the GLAST mission and to active galaxies, background information on the science and math in the activity, instructions on its use, and a student worksheet for classroom use. At the bottom is a brief overview of the E/PO program at Sonoma State university, the responsible people, a list of resources for further research, and a list of the national educational standards covered by this activity.

What is GLAST?
The Gamma-ray Large Area Space Telescope is a NASA satellite planned for launch in 2006. GLAST is part of NASA’s Structure and Evolution of the Universe theme. The astronomical satellites in this theme are designed to explore the structure of the Universe, examine its cycles of matter and energy, and peer into the ultimate limits of gravity: black holes. GLAST detects gamma rays, the highest energy light in the electromagnetic spectrum. GLAST is being built in collaboration between NASA, the U.S. Department of Energy, France, Germany, Italy, Japan, and Sweden. The project is managed from NASA’s Goddard Space Flight Center in Greenbelt, Maryland.

What instruments will GLAST use?
There are two scientific instruments on board GLAST: the Large Area Telescope (LAT) and the GLAST Burst Monitor (GBM). The LAT is the primary instrument, and will survey the sky in high-energy gamma-ray light. It will also study many individual sources of gamma rays. The GBM will detect Gamma Ray Bursts, tremendous explosions coming from vast distances. These explosions are thought to signal the birth of black holes.

Where do gamma rays come from?
Gamma ray sources include black holes, pulsars, supernova remnants and active galaxies. Active galaxies – distant galaxies with supermassive black holes in their cores – will be a very common source of high-energy gamma rays detected by GLAST. Thousands of these sources will be studied during its mission.

What will my students learn from these activities?
This is one of a series of activities that uses active galaxies as an engagement to teach basic concepts in physical science and mathematics. An introduction to active galaxies is on the next page, and a detailed listing of the applicable national science standards can be found below.

How are these activities organized?
The next three pages have a general introduction to active galaxies and the small angle formula. Depending on the age and ability of your students, you may need to tell them about this information, have them read it, or have a few advanced students put together a presentation to the rest of the class based on this introduction and other sources they may find. With this introduction, try to convey the excitement of the scientists when they first discovered interesting phenomena related to active galaxies, such as the super-massive black holes at their cores, and the high-energy jets of particles that are sometimes emitted.
Overview

The primary goal of the GLAST mission is to study Active Galaxies (or AG for short); distant galaxies with supermassive black holes in their cores. The activity on this poster uses the physical characteristics of active galaxies such as size and distance to teach students basic concepts in physical science and mathematics. The students first investigate how a classmate’s apparent size depends on their height and distance, and then use what they have learned to investigate the physical properties of active galaxies using the images on the poster.

Introduction to Active Galaxies

A galaxy is a system of stars, gas, and dust bound together by their mutual gravity. A typical galaxy has billions of stars, and some have trillions. Although they come in many different shapes, the basic structure is the same: a dense core of stars called a ‘nucleus’ surrounded by stars and gas. Normally, the core of a disk or elliptical galaxy is small, relatively faint, and composed of older, redder stars. However, in some galaxies the core is intensely bright, shining with power equivalent to trillions of suns, easily outshining the rest of the light of the galaxy combined. A galaxy that emits such tremendous amounts of energy is called an active galaxy. Active galaxies are actually rare compared to “normal” galaxies like our own Milky Way, but so bright they can be seen clear across the visible universe.

It is believed that at the center of these bright galaxies lies a supermassive black hole millions or even billions of times the mass of our Sun (Figure 1). As matter falls toward the black hole, it forms an accretion disk, a flattened disk of material swirling around the black hole. Friction and magnetic forces inside the disk heat it to millions of degrees, and it glows brightly nearly all the way across the electromagnetic spectrum, from radio waves up to X-rays. Although our Milky Way Galaxy has a central supermassive black hole, it is not an active galaxy. For reasons currently unknown, the black hole at the center of our Galaxy is quiescent, or inactive, as are most present-day galaxies.

Although the physics underlying the phenomenon is not well-understood, it is known that in some cases, the accretion disk focuses long jets of matter which streak away from the core at speeds near that of light. The jets are highly collimated (meaning they retain their narrow focus over vast distances) and are emitted in a direction perpendicular to the disk. Eventually, they slow to a stop due to friction with gas well outside the galaxy, forming giant clouds of matter that radiate strongly at radio wavelengths (radio lobes; Figure 2). In addition, surrounding the accretion disk is a torus (donut) of molecular material. From certain viewing angles, this torus can obscure observations of the black hole and accretion disk.

There are many types of active galaxies. Initially, when astronomers were first studying them, it was thought that the different types of AGs were fundamentally different objects. Now astronomers generally (but not universally) accept the unified model of AGs, meaning that most or all AGs are actually just different versions of the same object. Many of the apparent differences between types of AGs are due to viewing the AG at different orientations with respect to the disk, or due to observing the AG in different wavelengths of light.

Fig. 1 Artist’s illustration of an active galaxy

Fig. 2 Double-lobed radio galaxy NGC 4261
radio lobes that span some 100,000 light years out from a spiral-shaped disk of gas and dust 400 light years in diameter. Presumably, a small but supermassive black hole “engine” lies at the center of the nucleus with a mass 1.2 billion times the mass of the sun and contained in a space about the size of the solar system - about 6 billion kilometers. Imagine that!

How much detail can we actually see with the Hubble Space Telescope (HST) in a galaxy 100 million light years away?

The smallest objects HST can discern have an angle of about 0.1 arcseconds (about 0.00005 times the width of the full Moon). Using the small angle formula (see activity below) this angular diameter translates into 460 trillion kilometers at the distance of NGC 4261! This is over 100,000 times larger than the effective size of the black hole. That’s why we can’t directly see the black hole at the core of the galaxy. Rather, we infer the existence and the properties of the black hole indirectly by observing the effect it has on the gas and stars surrounding it.

The GLAST observatory, designed to detect gamma rays from active galaxies, will have a resolution of roughly 0.5 arcminutes (1/60th the width of the full Moon). Although no current detection technology exists that can see the black hole in the core of an active galaxy, GLAST will be able to detect gamma rays produced by the jets with unprecedented sensitivity. Such data will help us understand the physical processes that are going on in the nucleus and the supermassive black hole engine that fuels it.
Additional Background:

We imagine the sky to look like a hollow sphere surrounding the Earth. It really isn’t, but real objects are so far away that we have no real perception of distance. When we look at stars in the sky our brain interprets them as being infinitely far away. This gives the sky an illusion of being a hollow sphere with us at the center.

When astronomers measure the size of objects in the sky or the distance between them, they use **angular size**. A circle drawn all the way around the sky is divided into 360 degrees, so two objects on opposite sides of the sky are said to be 180 degrees apart. An object that spans ¼ of the sky’s circumference would be 90 degrees across, and so on. The disk of the Moon in these units has an angular diameter of 0.5 degrees. Degrees are further subdivided into 60 arcminutes (the term “arc” is to help distinguish this from a measure of time, and the unit is abbreviated as a single tick mark: ’) and each arcminute is further divided into 60 arcseconds (abbreviated with a double tick mark: ”). The Moon can thus be described as having an angular diameter of 30 arcminutes, or 1800 arcseconds. Most distant astronomical objects such as galaxies are fractions of an arcsecond to a few arcminutes in angular extent.

The relationship between angular diameter, distance, and appearance is shown below: the closer an object is, the larger the angle it covers and the larger it appears; the farther away an object is, the smaller the angle it covers and the smaller it appears.

Mathematically, angular diameter, linear diameter, and distance can be combined in an extremely useful and simple equation called the **small angle approximation**. As seen in the figure below, the angular diameter, \( \alpha \), depends on the distance to the object, \( D \), and its actual linear diameter, \( d \), according to:

\[
\tan(\alpha/2) = \frac{d}{2D}
\]

It can be shown that for very small values of \( \alpha \) (measured in radians), \( \tan(\alpha) = \alpha \). Using this approximation, the equation relating distance and linear size simplifies further to:

\[
\frac{\alpha}{2} = \frac{d}{2D} \quad \text{or simply} \quad \alpha = \frac{d}{D}
\]

In the small angle approximation, if any two of the quantities are known, the third can be calculated. In astronomy, the angular diameter is usually measured directly and the equation is used to calculate the distance or the physical diameter of the object. Since distances to astronomical objects are usually much larger than their linear sizes, this approximation is of great use in all branches and at all levels of astronomy!

The following table provides a summary of the unit conversions discussed above, along with some other abbreviations that will be used later in the unit. It may be useful for your students to have access to this table.
Materials for each group of 3 students:

- GLAST Active Galaxies Poster
- protractor
- stiff cardboard
- meter stick or tape measure
- pencil
- scissors
- calculator

Objectives:

Students will be able to construct a template and use it to correctly measure the angular size of a person.
Students will be able to use the Active Galaxies Poster to measure the angular size of a galaxy.

Procedure:

1. Introduce the activity by reviewing the information in the *Introduction to Active Galaxies* and the activity *Background Information*.

2. Discuss these questions with the students: Can they recognize their friend’s face from across the room? How about from across a football field? How far away can someone be and still be recognizable?

3. Explain to the students that they will be doing two exercises that will teach them how the angular size of objects changes with distance. They can do the exercises in teams of two or three people each.

4. For Part B, “Measuring the Size of a Galaxy Using the Active Galaxy Poster,” you will need to affix the poster to a wall, and make sure the students have about 5 meters of clear space between them and the poster. You can mount the poster at the end of a hallway or on the wall of a large classroom.

Assessment:

There are two separate activities that comprise this unit. They can be assessed using the common rubric below either individually or as a group.

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>All calculations are correct</td>
</tr>
<tr>
<td>3</td>
<td>One calculation is incorrect, or off by more than 10%</td>
</tr>
<tr>
<td>2</td>
<td>Two calculations are incorrect</td>
</tr>
<tr>
<td>1</td>
<td>More than two calculations are incorrect</td>
</tr>
<tr>
<td>0</td>
<td>No work turned in</td>
</tr>
</tbody>
</table>
Transfer Activities:

Students can test the limits of their vision and measure angles of familiar objects. For example, the width of a finger held at arm’s length is typically 1–2 degrees. This can be calibrated by having them mark their angle template in degrees from 1–10, and using it to measure the width of their fingers held at arm’s length. By comparing the known size of an object (say, the length of a car) to the width of their finger, they can approximate the distance to the object using the small angle formula.

Extension Activities:

Derive the small angle formula more rigorously, following the rules of trigonometry. Given that the Moon is 3500 km in diameter and 0.5º in angular size, have the students calculate the distance to the Moon. Have them look up the correct answer and compare it to their calculated answer.

Reflection Activities:

Where else would the small angle formula be useful? Examples: in surveying, hunting, sports, or anywhere else where gauging the distance would be necessary.

Lesson Adaptations:

If a student is visually impaired, and cannot measure the angles using the template in this activity, s/he can be the person whose height is measured, or can use the meter stick to measure distances. If the student is mobility-impaired, s/he can be the person whose height is measured (Student B).

Answers: Questions 1-6 depend on the students’ heights, so you must check the answers yourself, using the table below as a reference. [Note: In this table, the distances are given to 4 significant digits. You may wish to ask for a different approximation from your students.]

<table>
<thead>
<tr>
<th>Student Height (cm)</th>
<th>Distance (cm) for 0.17 radians=10º</th>
<th>Distance (km) for 0.0001 radians = 0.5 arcmin</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>802.1</td>
<td>9.626</td>
</tr>
<tr>
<td>150</td>
<td>859.4</td>
<td>10.31</td>
</tr>
<tr>
<td>160</td>
<td>916.7</td>
<td>11.00</td>
</tr>
<tr>
<td>170</td>
<td>974.0</td>
<td>11.69</td>
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<td>180</td>
<td>1031</td>
<td>12.38</td>
</tr>
<tr>
<td>190</td>
<td>1089</td>
<td>13.06</td>
</tr>
<tr>
<td>200</td>
<td>1146</td>
<td>13.75</td>
</tr>
</tbody>
</table>

Questions 7-20:

7. On average, the typical human eye can see objects about 1/60º of a degree across, so the answer to this question is “no.”
8. 13.0 centimeters. Note the significant figures should reflect 0.1 cm accuracy.
9. The distance should be 149 cm. This will depend on their measuring accuracy.
10. This will depend on their distance measurement, but should be close to the actual disk size of 13 cm.
11. This will depend on their accuracy. They should be within 10% or so of the measured size.
12. 893.8 meters.
13. 17.5 centimeters. Note the significant figures should reflect 0.1 cm accuracy.
14. The distance should be 200.5 cm, and will depend on their measuring accuracy.
15. This will depend on their distance measurement, but should be close to the actual lobe size of 17.5 cm.
16. This will depend on their accuracy. They should be within 10% or so of the measured size.
17. 1203.2 meters.
18. 446.9 meters.
Introduction to the Small Angle Approximation

In this activity, you will use a formula called “the small angle approximation” in order to determine the physical sizes of distant objects. The small angle approximation states: For small enough angles, the tangent of an angle is equal to the angle itself (when measured in radians). Or: \( \tan \alpha = \alpha \), where \( \alpha \) is the angle that you are measuring.

Astronomers use the small angle approximation to determine the sizes of distant objects. If the angular size of the object is called \( \alpha \), the distance is \( D \), and the diameter \( d \), then they are related by the equation

\[
\alpha = \frac{d}{D}
\]

For this formula to work, the angle must be measured in radians, an angular unit similar to degrees. Radians are related to the radius a circle (hence the name). The circumference of a circle is \( 2\pi \) times the radius. A radian is defined such that there are \( 2\pi \) of them in a circle, just like there are 360 degrees in a circle, so \( 2\pi \) radians = 360 degrees. Therefore, to convert degrees to radians, use the equations below:

\[
\text{(angle in degrees)} \times \frac{2\pi}{360} = \text{angle in radians}
\]

-or-

\[
\text{(angle in degrees)} \times 0.01745 = \text{angle in radians}
\]

In the following exercise, you will measure the apparent size of a distant object in degrees using a template. You will need to convert the size to radians to calculate the real, physical size of the object.

A. Measuring the Angular Size of a Person

First, construct a template that measures a 5° and a 10° angle to use in the exercise. Place your stiff piece of cardboard in front of you so that one of the long edges is nearest you. Mark a point near the lower right hand corner on that long edge.

Place your protractor so that the hole in the bottom edge of the protractor is centered on the mark on the cardboard. First, measure a 5° angle going off to the left hand side and mark it. Using a straightedge, connect the first mark to the second, creating a 5° angle with the bottom edge of the cardboard. Make the line as long as possible, and draw it dark enough to see well.

Now label the angle. Draw a little arc going from the bottom of the template up to the 5° line. Next to it, write “5°”. Then convert that angle to radians and write that number next to where you wrote “5°”, so it says “5° = X radians”, replacing the X with the number you calculated.

Next, repeat this procedure using a 10° angle. You should now have two lines, one at 5° and the other at 10°, starting at the lower right corner of the paper and going toward the upper left. Using scissors, carefully cut the cardboard along the 10° angle.

Make sure you cut the vertex of the angle carefully! If the narrow tip of the angle gets cut off, your measurements will be off. When you are done, you should have a long, narrow triangle. The entire angle is 10°, and it should be bisected by a dark line running along it that measures a smaller 5° angle.
You can check the accuracy of your template by measuring the 10° angle with your protractor again. It should be as close to 10° as possible, but no more than 0.5° off. If it’s off by more than that, you’ll need to either trim your template or make a new one.

Pick the roles each team member will perform in this activity: Student A will be using the template to measure angles, Student B will be measured, and Student C will be the measurer.

Student C: Using the meter stick, carefully measure the height of Student B to within a centimeter.

**Question 1: Height of Student B:**

__________ (cm)

Student A will now measure the angular size of Student B. Make sure you have enough room to do this! You’ll need about 8-15 meters between them, so you may have to do this in a hallway or outside.

Students A and B: Start off by standing next to each other.

Student C: Mark the position of Student A on the floor/ground. (The mark should represent where Student A’s eyes are, and not toes! Gauge where Student A’s eyes are over the oor, or simply mark where the ankle is, which is roughly under the eye.)

Student B: Start walking away from Student A.

Student A: Using the angle template, compare the size of Student B to the 10° angle on the template. (The best way to do this is to pinch the narrow end of the angle with your thumb and index finger, and hold it up to your face on the outside of your eye. That way, the vertex of the angle is aligned with your eye.) When Student B has walked far enough away that s/he appears to be the same size as the 10° angle, tell Student B to stop. Student B may need to move a bit closer or farther to adjust his or her angular size to match the template. If Student B appears smaller than the angle, tell Student B to move closer to you until Student B appears to be the same size as the end of the angle measure. If Student B appears too large, tell Student B to move away. Match the angle as carefully as you can. Remember, mark the floor under the eyes, not the toes!

Student C: When Student B is at the right distance, mark this position.

Student C: With the meter stick or tape measure, measure the distance between Student A and Student B, to the nearest centimeter.

**Question 2: Measured distance:**

__________ (cm)

Calculate the height of Student B using the small angle formula: \( \alpha = \frac{d}{D} \). In this equation, \( \alpha \) is the 10° angle (but in radians!) that you used to position the student, lower case \( d \) is the height of Student B and upper case \( D \) is the distance that you have measured in question 4.

**Question 3: Calculated height:**

__________ (cm)

How close were you able to calculate the actual height? Subtract the calculated value from the measured value.

**Question 4: Difference in height:**

__________ (cm)

Calculate this difference as a percent:

The percent difference in distance =

\[
\frac{(\text{measured} - \text{calculated})}{\text{(measured)}} \times 100
\]

**Question 5: Percent difference:**

__________

How far away would Student B have to stand from Student A in order to have an angular measure of 0.5 arcminutes, the approximate resolution of GLAST? Remember there are 60 arcminutes in a degree, and you must convert to radians. Use the small angle formula to express your answer in kilometers to the nearest 10 meters (0.01 km).

**Question 6: Distance for 0.5 arcminutes:**

__________ (km)

**Question 7: Do you think you would still be able to see Student B as more than a dot with your unaided eye from that distance?**

__________
B. Measuring the Angular Size of a Galaxy Using the Active Galaxies Poster

**Student C:** With a meter stick or metric ruler, measure the diameter of the disk along its longest dimension using the middle picture on the left of the poster. Measure to the nearest 0.1 centimeters.

**Question 8:** AG disk size: ______________________ (cm)

**Student A:** Move away from the poster until the disk in the middle left panel of the poster subtends an angle of 5° as measured with the cardboard angle template.

**Student C:** Mark the spot on the floor as in the previous exercise, and measure the distance from Student A to the poster.

**Question 9:** Measured disk distance (5°): ______________________ (cm)

Using your answer from Question 9 and the small angle formula, calculate the size of the disk to the nearest 0.1 centimeters.

**Question 10:** Calculated disk size (5°): ______________________ (cm)

How accurate was your measurement? Calculate the percent difference between the measured and calculated size of the disk.

**Question 11:** Percent difference in disk sizes: ________________

Using the small angle formula, calculate how far you would have to stand from the poster so that the disk would subtend 0.5 arcminutes - the resolution of the GLAST telescope. Express your answer in meters.

**Question 12:** Disk distance (0.5°): ______________________ (m)

**Student C:** With a meter stick or metric ruler, measure the radio lobe span of the AG in the upper left corner of the poster, from radio lobe tip to radio lobe tip.

**Question 13:** AG radio lobe size: ______________________ (cm)

Find the distance from the poster so that the radio lobes (from tip to opposite tip) subtend an angle of 5°.

**Question 14:** Measured radio lobe distance (5°): ______________________ (cm)

Using your answer from Question 14 and the small angle formula, calculate the size of the radio lobes to the nearest 0.1 cm.

**Question 15:** Calculated radio lobe size (5°): ______________________ (cm)

Calculate the percent difference in your measured versus calculated sizes for the radio lobes.

**Question 16:** Percent difference in radio lobe sizes: ________________

How far would you have to stand from the poster so the radio lobes subtend 0.5 arcminutes? Express your answer in meters.

**Question 17:** Radio lobe distance (0.5°): ______________________ (m)

What are the limits of your own vision? The average human eye can just barely distinguish two objects that are 1 arcminute apart. (Your own vision may vary from this.) Using the small angle formula, determine how far away the poster would have to be in order for you to barely see the disk as more than a dot. Express your answer in meters.

**Question 18:** Limit Distance: ________________ (m)

Look again at your answer for Question 7. Did you answer the question correctly?
For more information on GLAST and active galaxies, see the GLAST Education and Public Outreach web site:
http://glast.sonoma.edu

This educational unit can be found on the GLAST Education and Public Outreach web site at:
http://glast.sonoma.edu/teachers/teachers.html
For a copy of the GLAST Active Galaxies Poster, and activity booklet, please contact:
materials@universe.sonoma.edu

The Center for Astrophysical Research in Antarctica has an interesting web page with small angle formula activities:
http://astro.uchicago.edu/cara/outreach/resources/ysi94/smallangle.html

Bill Keel’s Active Galaxies page:
http://www.astr.ua.edu/keel/agn/

Active Galaxies from the University of Maryland:

Ted Bunn’s Black Hole tutorial:
http://cosmology.berkeley.edu/Education/BHfaq.html

Compton Gamma-Ray Observatory and EGRET:
http://cossn.gsfc.nasa.gov/index.html

GLAST Project Site at Goddard Space Flight Center: http://glast.gsfc.nasa.gov/
GLAST Large Area Telescope (LAT) Collaboration: http://www-glast.stanford.edu/
GLAST Burst Monitor (GBM): http://gammaray.msfc.nasa.gov/gbm/
This activity addresses the following National Science Education Content Standards for grades 9-12:

**Content Standard A: Science as Inquiry**  
*Abilities necessary to do scientific inquiry*

- Using their own calculations, the students formulate and revise the theory about an object’s size.
- After having analyzed measurements of nearby objects, students answer questions that engage thought and analysis about real objects in space.

*Understanding about scientific inquiry*

- Students learn how scientists determine the distance and/or size of an object in space.

**Content Standard B: Physical Science**  
*Motion and Forces*

- Jets of materials are ejected at velocities near light speed from the black hole in an AG. Students answer questions to help them see how big the jets from AGs are, and how we can see them at vast distances.

**Content Standard D: Earth and Space Science**  
*Origin and Evolution of the Universe*

- Active galaxies are a fundamental part of the evolutionary process of the universe.

**Content Standard E: Science and Technology**  
*Understanding about science and technology*

- The small angle formula is an essential tool used by astronomers to get physical dimensions of astronomical objects.

**Content Standard G: History and Nature of Science**  
*Science as a human endeavor*

- Students answer questions about the ability of the human eye to distinguish objects, showing how this activity affects them in daily life.
- Students see that by working in groups they can formulate better hypotheses about scientific inquiries due to the extra input from others.
**Accretion Disk:** the flattened disk of matter swirling just outside the black hole.

**Active Galaxy:** a galaxy with an unusually large amount of energy emitted from the nucleus.

**Angular Size:** the angle between the lines of sight to the two opposite sides of an object.

**Black Hole:** an object so small and dense that inside its event horizon, the escape velocity is faster than the speed of light. In an active galaxy, the central black hole may have millions or even billions of times the Sun’s mass.

**Jet:** a thin, highly focused beam of matter and energy emitted from the nuclei of some active galaxies. Jets can be hundreds of thousands of light years long.

**Quiescent:** at rest; inactive.

**Radio Lobe:** A large radio-wave emitting cloud of matter located at the ends of the jets in some active galaxies, formed when the matter from the jet is slowed by intergalactic material.

**Torus:** A doughnut-shaped object. Gas and dust outside the accretion disk in an active galaxy orbit the central black hole in a torus-shaped region.

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**Who developed these activities?**
The activities and the poster that describes Active Galaxies have been developed as part of the NASA Education and Public Outreach (E/PO) Program at Sonoma State University, under the direction of Professor Lynn Cominsky. The poster is available by request at [http://glast.sonoma.edu/teachers/teachers.html](http://glast.sonoma.edu/teachers/teachers.html).

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